# Finite Longitudinally Multilayered Membrane Shells Subjected to Impact Loads

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## Theme

THE transient response is obtained for a finite longitudinally multilayered cylindrical membrane shell impacted by longitudinal axisymmetric loads. The shell is composed of layers running lengthwise where each layer is made of a different isotropic material. Membrane stress distribution and velocities of particles are determined for the entire solution domain. The method of solution consists of a characteristics finite difference technique and discontinuity relations. A characteristics finite difference integration scheme is constructed for regions where the variables are continuous. Discontinuity relations are developed for an impulsive leading wave and its reflection and refraction at the interfaces. A problem is solved demonstrating this method. This paper is an extension of a previous work where the composed shell was confined to two special layers. The present generalized analysis is valid for any number of layers, unrestricted to length or material properties.

## Contents

The analysis presented here is based on the method of characteristics. Applications of this method to semi-infinite homogeneous shell problems are shown in Refs. 1–3. For the finite longitudinally multilayered shell the phenomenon of reflection and refraction taking place at the interfaces of the layers is obtained here separately for the leading wave and for the remainder solution domain. For this algorithm a two-layered shell problem was solved in Ref. 4. However, the two layers were chosen in a special manner in order to form a continuous characteristics grid for the entire solution domain. The objective of this program is to provide a generalized technique where the different characteristics grids do not necessarily match. Thus, any multilayered shell is considered where the layers are not confined to length nor to material properties.

The characteristics formulation and the discontinuity relations across an impulsive leading wave are presented by Eqs. (1–7) of Ref. 4. Since no fracture is allowed and since reaction equals absolute action the equations governing the reflection and refraction of discontinuities at the interface are derived and are given here in dimensionless form, as follows:

$$[W] = O,$$

$$[U] = -\frac{([N_x]-\alpha_i[U]) \text{ given from the incoming leading wave}}{\alpha_i+\alpha_{i+1}}$$

$$[N_x] = -\alpha_i[U], \quad [N_{\theta i}] = \nu_1\nu_i[N_x]$$

and

$$[N_{\theta_{i+1}}] = \nu_1\nu_{i+1}[N_x]$$

By a similar approach the discontinuities at the end point of the leading wave are evaluated from the following relations:

$$[N_x] = [N_\theta] = [W] = O$$

and

$$[U] = -\frac{1}{\alpha_k}\{[N_x]-\alpha_k[U]\} \text{ given from the incoming wave}$$

where the subscript 1 refers to the material properties of the first layer, $i$ represents the specific layer, and $k$ the last layer. Once the discontinuity relations for the leading wave and for its reflection and refraction are available the motion behind the leading wave is determined by the characteristic formulation. A finite difference integration scheme is proposed for the characteristic equations along the characteristic lines for nine typical points as depicted by Fig. 1.

An initial boundary-value problem is considered where the developed scheme is demonstrated. The problem consists of a five-layered cylindrical membrane shell subjected to an impulsive axisymmetric load applied at one end of the composed finite shell. The applied load is an exponential overpressure and is presented in the following form:

$$n_x = n_{x_o} e^{-\psi t}$$

or in dimensionless form

$$N_x = N_{x_o} e^{-\psi R\tau/G_{L_1}}$$

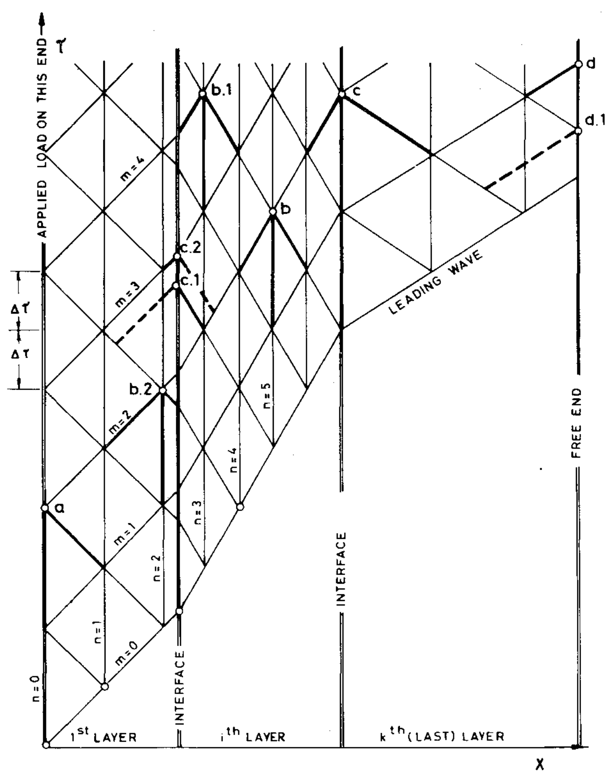


Fig. 1 Integration scheme and nine typical points.

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Index category: Structural Dynamic Analysis.

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**Table 1 The composed shell**

| Layer number $i$ | Material | $\rho \times 10^{-4}$ $\frac{\text{lb}_f\text{-sec}^2}{\text{in.}^4}$ | $E \times 10^6$ psi | $\nu$ | $G_L \times 10^5$ in./sec | $L$ in. | $R$ in. | $h$ in. | $\bar{L} = L/R$ | Acoustic impedance $\rho G_L \frac{\text{lb-sec}}{\text{in.}^3}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Aluminum | 2.54 | 10.2 | 0.31 | 2.108 | 3 | 10 | 0.1 | 0.3 | 53.6 |
| 2 | Copper | 8.33 | 17.4 | 0.34 | 1.537 | 4 | 10 | 0.1 | 0.4 | 128 |
| 3 | Steel | 7.28 | 30.5 | 0.29 | 2.137 | 6 | 10 | 0.1 | 0.6 | 156 |
| 4 | Copper | 8.33 | 17.4 | 0.34 | 1.537 | 7 | 10 | 0.1 | 0.7 | 128 |
| 5 | Aluminum | 2.54 | 10.2 | 0.31 | 2.108 | 9.186 | 10 | 0.1 | 0.9186 | 53.6 |

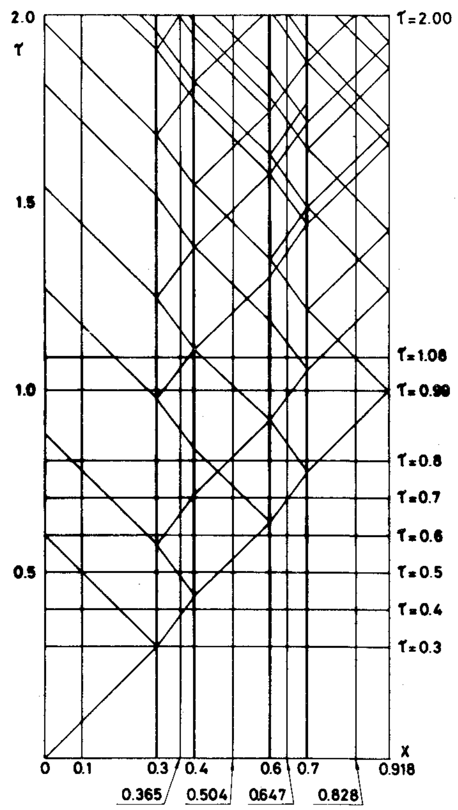


**Fig. 2 Wave propagation partial chart.**

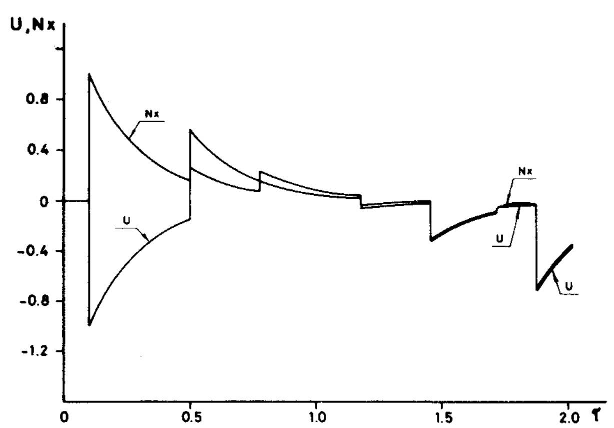


**Fig. 3 $N_x$ and $U$ at $x = 0.1$.**

where $N_{x_0}$ is the peak magnitude taken to be unity and $\psi$ is a damping factor taken to be $10^5$ 1/sec for this example. The geometry and material properties are given in Table 1.

The study presented a method for the solution of impulsive stress wave propagation problems in layered shells. It can be concluded that the method is directly applicable to any number of longitudinal layers forming a membrane type shell. A five-layered shell impacted by a longitudinal axisymmetric load was solved in details. In order to follow the propagation of waves in the layered shell a chart was laid out (Fig. 2) where the leading, reflected, and refracted waves were depicted with respect to space and time. Figures 3 and 4 were plotted according to this chart to show the resulted motion in the impacted layered shell. In view of the results it can be concluded that the reflection and refraction "jumps" of the longitudinal stress resultant and the longitudinal velocity are proportional to the acoustic impedance $\rho G_L$. The "jumps" of the circumferential stress resultant $N_\theta$ are due to "jumps" of $N_x$ and proportional to the Poisson ratio. The radial velocity $W$, as opposed to the other dependent variables, varies in a continuous manner in each layer.

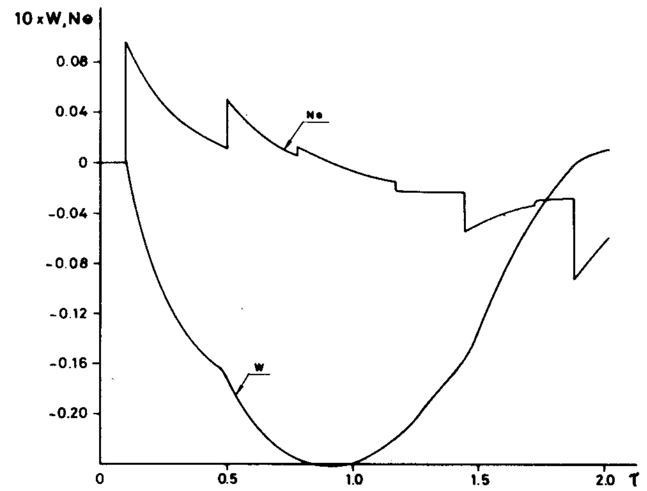


**Fig. 4 $N_\theta$ and $W$ at $x = 0.1$.**

Effects of layers in a shell on its dynamic deformation are studied in this paper. Reinforced shell design must take into account these effects in order to optimize the structure. In general, axial impacts result in quasilongitudinal waves which have a significant axial bending component.[3] It is known, however, that cylindrical shells impacted axially will support pure longitudinal waves when $R/h$ is large or $L/R$ is small. The present analysis is confined to a large $R/h$.

## References


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